Physics II
ISI B.Math
Back Paper Exam
January 7, 2015

Total Marks: 50

Answer all questions

1. **(Marks : 4 + 6 = 10)**

(a) Use the fact that the entropy S is a state function and dS is an exact differential to show that

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

Hence show that the internal energy of an ideal gas is a function of temperature only.

(b) One mole of an ideal gas obeys the van der Waals equation of state.

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

where a and b are constants. Find the molar internal energy and entropy, given that the molar heat capacity C at constant volume is a constant.

2. **(Marks : 3 + 3 + 4 = 10)**

An imaginary ideal gas engine operates in a cycle, which forms a rectangle with sides parallel to the axes of a $P - V$ diagram. Call P_1 and P_2 the lower and higher pressures respectively; call V_1 and V_2 the lower and higher volumes respectively.

(a) Calculate the work done in one cycle.

(b) Indicate which parts of the cycle involve heat flow into the gas, and calculate the amount of heat flowing into the gas. (Assume constant heat capacities C_V and C_P and $\gamma = \frac{C_p}{C_V}$)

(c) Show that the efficiency of this heat engine is

$$\eta = \frac{\gamma - 1}{\frac{\gamma P_2}{P_2 - P_1} + \frac{V_1}{V_2 - V_1}}$$

3. **(Marks : 7 + 3 = 10)**

Consider a model system of N spins at equilibrium at temperature T. Each spin carries a magnetic moment that can assume the value $+m$ or $-m$. The energy of each spin in a magnetic field B is $-mB$ when the spin points along the magnetic field and mB when it points opposite to the magnetic field.

(a) Use the partition function to find an exact expression for the magnetization (the magnetic moment per unit volume)M and the susceptibility $\chi = \frac{dM}{dB}$ as a function of the temperature and the magnetic field.

(b) Show that the susceptibility is $\chi = \frac{nm^2}{kT}$ in the limit $mB \ll kT$ where n is the number of particles per unit volume and that it vanishes in the limit $mB \gg kT$.

4. **(Marks : 5 + 5 = 10)**

(a) A plane monochromatic light wave in air falls at normal incidence on a thin film of oil which covers a glass plate, The wavelength of the source may be varied continuously. Complete destructive interference is observed for wavelengths of 5000 and 7000 A and for no wavelengths in between. Show that the index of refraction for the oil must be less than 1.50.

(b) In Young's double slit experiment, a thin mica sheet (n = 1.5) is introduced in the path of one of the beams. If the central fringe gets shifted by 1.2 cm, calculate the thickness of the mica sheet, Assume that the separation between the slits is 0.1 cm and the distance between the slits and the screen is 50 cm.

5. **(Marks : 3 + 3 + 4 = 10)**

(a) What should be the requirements on the slit separation d and the slit width a, such that the central maximum of the envelope of the double slit Fraunhofer diffraction pattern contains exactly 11 fringes ?

(b) What is the state of polarization of a light wave when the x and y components of the electric field are given by the following equations $E_x = E_0 \cos(\omega t + kz), E_y = E_0 \cos(\omega t + kz + \pi)$. Justify your answer.

(c) A diffraction grating 2.0 cm width has 6000 rulings. At what angles will the maximum intensity beams occur if the incident radiation has a wavelength of 5600A ?